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μ-τ Symmetry and Maximal CP Violation

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We argue the possibility that a real part of a flavor neutrino mass matrix only respects a μ-τ symmetry. This possibility is shown to be extended to more general case with a phase parameter θ, where the μ-τ symmetric part has a phase of $\theta/2$. This texture shows maximal CP violation and no Majorana CP violation.

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The present experimental data on neutrino oscillations [1, 2] indicate the mixing angles [3] satisfying

$$0.70 < \sin^2 2\theta_\odot < 0.95, \quad 0.92 < \sin^2 2\theta_{atm}, \quad \sin^2 \theta_{CHOOZ} < 0.05, \tag{1}$$

where $\theta_\odot$ is the solar neutrino mixing angle, θ_{atm} is the atmospheric neutrino mixing angle and θ_{CHOOZ} is for the mixing angle between ν_e and ν_τ. These mixing angles are identified with the mixings among three flavor neutrinos, ν_e, ν_μ and ν_τ, yielding three massive neutrinos, $\nu_{1,2,3}$: $\theta_{12} = \theta_\odot$, $\theta_{23} = \theta_{atm}$ and $\theta_{13} = \theta_{CHOOZ}$. These data seem to be consistent with the presence of a μ-τ symmetry [4, 5, 6, 7] in the neutrino sector, which provides maximal atmospheric neutrino mixing with $\sin^2 2\theta_{23} = 1$ as well as $\sin\theta_{13} = 0$.

Although neutrinos gradually reveal their properties in various experiments since the historical Super-Kamiokande confirmation of neutrino oscillations [1], we expect to find yet unknown property related to CP violation [8]. The effect of the presence of a leptonic CP violation can be described by four phases in the PMNS neutrino mixing matrix, U_{PMNS} [9], to be denoted by one Dirac phase of δ and three Majorana phases of $\beta_{1,2,3}$ as $U_{PMNS} = U_\nu K$ [10] with

$$\begin{aligned} U_\nu &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta} & c_{23}c_{13}, \end{pmatrix}, \\ K &= \mathrm{diag}(e^{i\beta_1}, e^{i\beta_2}, e^{i\beta_3}), \end{aligned} \tag{2}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$ (i, j=1,2,3) and Majorana CP violation is specified by two combinations made of $\beta_{1,2,3}$ such as $\beta_i - \beta_3$ in place of β_i in K. To examine such effects of CP violation, there have been various discussions [11] including those on the possible textures of flavor neutrino masses [12, 13, 14, 15].

In this note, we would like to focus on the role of the μ-τ symmetry in models with CP violation [14, 15], which can be implemented by introducing complex flavor neutrino masses. The μ-τ symmetric texture gives $\sin\theta_{13} = 0$ as well as maximal atmospheric neutrino mixing characterized by $c_{23} = \sigma s_{23} = 1/\sqrt{2}$ ($\sigma = \pm 1$). Because of $\sin\theta_{13} = 0$, Dirac CP violation is absent in Eq.(2) and CP violation becomes of the Majorana type. Since the μ-τ symmetry is expected to be approximately realized, its breakdown is signaled by $\sin\theta_{13} \neq 0$. To have $\sin\theta_{13} \neq 0$, we discuss another implementation of the μ-τ symmetry such that the symmetry is only respected by the real part of M_ν. The discussion is based on more general case, where M_ν is controlled by one phase to be denoted by θ and the specific value of $\theta = 0$ yields the μ-τ symmetric real part. It turns out that Majorana CP violation is absent because all three Majorana phases are calculated to be $-\theta/4$ while Dirac CP violation becomes maximal.

Our complex flavor neutrino mass matrix of M_ν is parameterized by

$$M_\nu = \begin{pmatrix} M_{ee} & M_{e\mu} & M_{e\tau} \\ M_{e\mu} & M_{\mu\mu} & M_{\mu\tau} \\ M_{e\tau} & M_{\mu\tau} & M_{\tau\tau} \end{pmatrix}, \tag{3}$$

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where $U_{PMNS}^T M_\nu U_{PMNS}$=diag.$(m_1,\ m_2,\ m_3)$.[1] The mixing angles have been calculated in the Appendix of Ref.[16] and satisfy

$$\sin 2\theta_{12}\left(\lambda_1-\lambda_2\right)+2\cos 2\theta_{12}X=0, \tag{4}$$
$$\sin 2\theta_{13}\left(M_{ee}e^{-i\delta}-\lambda_3 e^{i\delta}\right)+2\cos 2\theta_{13}Y=0, \tag{5}$$
$$\left(M_{\tau\tau}-M_{\mu\mu}\right)\sin 2\theta_{23}-2M_{\mu\tau}\cos 2\theta_{23}=2s_{13}e^{-i\delta}X, \tag{6}$$

and neutrino masses are given by

$$m_1 e^{-2i\beta_1}=\frac{\lambda_1+\lambda_2}{2}-\frac{X}{\sin 2\theta_{12}},\quad m_2 e^{-2i\beta_2}=\frac{\lambda_1+\lambda_2}{2}+\frac{X}{\sin 2\theta_{12}},$$
$$m_3 e^{-2i\beta_3}=\frac{c_{13}^2\lambda_3-s_{13}^2 e^{-2i\delta}M_{ee}}{\cos 2\theta_{13}}, \tag{7}$$

where

$$\lambda_1=c_{13}^2 M_{ee}-2c_{13}s_{13}e^{i\delta}Y+s_{13}^2 e^{2i\delta}\lambda_3,\quad \lambda_2=c_{23}^2 M_{\mu\mu}+s_{23}^2 M_{\tau\tau}-2s_{23}c_{23}M_{\mu\tau},$$
$$\lambda_3=s_{23}^2 M_{\mu\mu}+c_{23}^2 M_{\tau\tau}+2s_{23}c_{23}M_{\mu\tau}, \tag{8}$$
$$X=\frac{c_{23}M_{e\mu}-s_{23}M_{e\tau}}{c_{13}},\quad Y=s_{23}M_{e\mu}+c_{23}M_{e\tau}. \tag{9}$$

To clarify the importance of the μ-τ symmetry, which accommodates maximal atmospheric neutrino mixing and $\sin\theta_{13}=0$, we first review what conditions are imposed by the requirement of $\sin\theta_{13}=0$. From Eq.(5), we require that

$$Y=s_{23}M_{e\mu}+c_{23}M_{e\tau}=0, \tag{10}$$

giving rise to $\tan\theta_{23}=-\mathrm{Re}(M_{e\tau})/\mathrm{Re}(M_{e\mu})=-\mathrm{Im}(M_{e\tau})/\mathrm{Im}(M_{e\mu})$. Since $\sin\theta_{13}=0$, Eq.(6) reads

$$\left(M_{\tau\tau}-M_{\mu\mu}\right)\sin 2\theta_{23}=2M_{\mu\tau}\cos 2\theta_{23}. \tag{11}$$

These are the well known relations that determine θ_{23} if $\sin\theta_{13}=0$. Maximal atmospheric neutrino mixing arises if

$$M_{\tau\tau}=M_{\mu\mu}, \tag{12}$$

which in turn gives

$$M_{e\tau}=-\sigma M_{e\mu}. \tag{13}$$

The flavor neutrino masses satisfying Eqs.(12) and (13) suggest the presence of μ-τ symmetry in neutrino physics. The remaining mixing angle of θ_{12} satisfies

$$M_{\mu\mu}-\sigma M_{\mu\tau}=M_{ee}+\frac{2\sqrt{2}}{\tan 2\theta_{12}}M_{e\mu}, \tag{14}$$

which determines the definite correlation of the phases of the flavor neutrino masses.

In place of Eqs.(10) and (11), using a Hermitian matrix of $\mathbf{M}=M_\nu^\dagger M_\nu$, we can find that $\tan\theta_{23}=-\mathrm{Re}\left(\mathbf{M}_{e\tau}\right)/\mathrm{Re}\left(\mathbf{M}_{e\mu}\right)=-\mathrm{Im}\left(\mathbf{M}_{e\tau}\right)/\mathrm{Im}\left(\mathbf{M}_{e\mu}\right)$, where $\mathbf{M}_{e\mu}=M_{ee}^*M_{e\mu}+M_{e\mu}^*M_{\mu\mu}+M_{e\tau}^*M_{\mu\tau}$ and $\mathbf{M}_{e\tau}=M_{ee}^*M_{e\tau}+M_{e\mu}^*M_{\mu\tau}+M_{e\tau}^*M_{\tau\tau}$. In addition, we have argued that $\tan\theta_{23}$ is directly determined by $\tan\theta_{23}=\mathrm{Im}(\mathbf{M}_{e\mu})/\mathrm{Im}(\mathbf{M}_{e\tau})$ satisfied in any models with complex neutrino masses irrespective of the values of $\sin\theta_{13}$ [17]. Both expressions of $\tan\theta_{23}$ are compatible if $(\mathrm{Im}(\mathbf{M}_{e\mu}))^2+(\mathrm{Im}(\mathbf{M}_{e\tau}))^2$=0, yielding $\mathrm{Im}(\mathbf{M}_{e\mu})=\mathrm{Im}(\mathbf{M}_{e\tau})=0$. Since the Dirac CP violation phase is absent for $\sin\theta_{13}=0$, $\mathbf{M}$ with the Majorana phases cancelled is necessarily real. In fact, we obtain that $\mathbf{M}_{e\mu}=c_{12}s_{12}c_{23}(m_2^2-m_1^2)$ and $\mathbf{M}_{e\tau}=-\tan\theta_{23}\mathbf{M}_{e\mu}$ which automatically satisfy $\mathrm{Im}(\mathbf{M}_{e\mu})=\mathrm{Im}(\mathbf{M}_{e\tau})=0$.

We next argue the implementation of the μ-τ symmetry based on the observation that it is sufficient for the symmetry to be respected by the real part of M_ν. From the discussions developed in Ref.[16], it can be extended to more general

[1] It is understood that the charged leptons and neutrinos are rotated, if necessary, to give diagonal charged-current interactions and to define the flavor neutrinos of ν_e, ν_μ and ν_τ.

case, where the real and imaginary parts are, respectively, replaced by $(z+e^{i\theta}z^*)/2$ ($\equiv z_+$) and $(z-e^{i\theta}z^*)/2$ ($\equiv z_-$) for a complex number of z and the phase parameter of θ. It is useful to notice that $z_+ = e^{i\theta/2}\mathrm{Re}(e^{-i\theta/2}z)$ and $z_- = ie^{i\theta/2}\mathrm{Im}(e^{-i\theta/2}z)$. The relevant mass matrix is provided by one of the textures found in Ref.[16]:

$$M_\nu = \begin{pmatrix} M_{ee} & M_{e\mu} & -\sigma e^{i\theta}M^*_{e\mu} \\ M_{e\mu} & M_{\mu\mu} & M_{\mu\tau} \\ -\sigma e^{i\theta}M^*_{e\mu} & M_{\mu\tau} & e^{i\theta}M^*_{\mu\mu} \end{pmatrix}, \tag{15}$$

where $M_{ee,\mu\tau} = e^{i\theta}M^*_{ee,\mu\tau}$, equivalently $(M_{ee,\mu\tau})_- = 0$, is imposed. This texture gives

$$\tan 2\theta_{12} = 2\sqrt{2}\frac{\cos 2\theta_{13}\left(M_{e\mu}\right)_+}{c_{13}\left[\left(1-3s^2_{13}\right)\left(M_{\mu\mu}\right)_+ - c^2_{13}\left(\sigma\left(M_{\mu\tau}\right)_+ + \left(M_{ee}\right)_+\right)\right]}, \tag{16}$$

$$\tan 2\theta_{13}e^{i\delta} = 2\sqrt{2}\frac{\sigma\left(M_{e\mu}\right)_-}{\left(M_{\mu\mu}\right)_+ + \sigma\left(M_{\mu\tau}\right)_+ + \left(M_{ee}\right)_+}. \tag{17}$$

As discussed in Ref.[16], these expressions yield real values of $\tan 2\theta_{12,13}$ because of the property that $z^\prime_+/z_+ = \mathrm{Re}(e^{-i\theta/2}z^\prime)/\mathrm{Re}(e^{-i\theta/2}z)$ and $z^\prime_-/z_+ = i\mathrm{Im}(e^{-i\theta/2}z^\prime)/\mathrm{Re}(e^{-i\theta/2}z)$ for any complex values of z and $z^\prime$. As a result, $\delta = \pm\pi/2$ is derived and M_ν gives maximal CP violation.

A texture with the Dirac CP violation phase related to the μ-τ symmetric texture is obtained by decomposing z and $e^{i\theta}z^*$ into z_+ and z_- and turns out to be $M_\nu = M_{+\nu} + M_{-\nu}$ with

$$M_{+\nu} = \begin{pmatrix} (M_{ee})_+ & (M_{e\mu})_+ & -\sigma(M_{e\mu})_+ \\ (M_{e\mu})_+ & (M_{\mu\mu})_+ & (M_{\mu\tau})_+ \\ -\sigma(M_{e\mu})_+ & (M_{\mu\tau})_+ & (M_{\mu\mu})_+ \end{pmatrix} = e^{i\theta/2}\mathrm{Re}\left(e^{-i\theta/2}M_\nu\right),$$

$$M_{-\nu} = \begin{pmatrix} 0 & (M_{e\mu})_- & \sigma(M_{e\mu})_- \\ (M_{e\mu})_- & (M_{\mu\mu})_- & 0 \\ \sigma(M_{e\mu})_- & 0 & -(M_{\mu\mu})_- \end{pmatrix} = ie^{i\theta/2}\mathrm{Im}\left(e^{-i\theta/2}M_\nu\right), \tag{18}$$

which shows that $M_{+\nu}$ has a phase $\theta/2$ modulo π while $M_{-\nu}$ has a phase $(\theta+\pi)/2$ modulo π. The μ-τ symmetry exists in $M_{+\nu}$ because Eqs.(12) and (13) are satisfied but is explicitly broken by $M_{-\nu}$. Therefore, this texture shows "incomplete" μ-τ symmetry [15]. Since $M_{+\nu}$ does not contribute to $\sin\theta_{13}$, $\sin\theta_{13}$ should be proportional to the flavor neutrino masses in $M_{-\nu}$. In fact, it is proportional to $(M_{e\mu})_-$ in Eq.(17). To speak of the Majorana phases, we have to determine neutrino masses, which can be computed from Eq.(7) and are given by

$$\begin{aligned} m_1e^{-2i\beta_1} &= \left(M_{\mu\mu}\right)_+ - \sigma\left(M_{\mu\tau}\right)_+ - \frac{1+\cos 2\theta_{12}}{\sin 2\theta_{12}}\frac{\sqrt{2}\left(M_{e\mu}\right)_+}{c_{13}}, \\ m_2e^{-2i\beta_2} &= \left(M_{\mu\mu}\right)_+ - \sigma\left(M_{\mu\tau}\right)_+ + \frac{1-\cos 2\theta_{12}}{\sin 2\theta_{12}}\frac{\sqrt{2}\left(M_{e\mu}\right)_+}{c_{13}}, \\ m_3e^{-2i\beta_3} &= \frac{c^2_{13}\left(\left(M_{\mu\mu}\right)_+ + \sigma\left(M_{\mu\tau}\right)_+\right) + s^2_{13}\left(M_{ee}\right)_+}{\cos 2\theta_{13}}. \end{aligned} \tag{19}$$

Since $z_+ = e^{i\theta/2}\mathrm{Re}(e^{-i\theta/2}z)$, the texture gives three Majorana phases calculated to be: $\beta_{1,2,3} = -\theta/4$ modulo $\pi/2$. The common phase does not induce Majorana CP violation. This result reflects the fact that the source of the Majorana phases is the phase of M_ν in Eq.(18) equal to $\theta/2$, which can be rotated away by redefining appropriate fields. The remaining imaginary part $\mathrm{Im}(e^{-i\theta/2}M_\nu)$ supplies the Dirac phase δ. Therefore, our proposed mass matrix becomes $\mathrm{Re}(e^{-i\theta/2}M_\nu) + i\mathrm{Im}(e^{-i\theta/2}M_\nu)$, which is equivalent to M_ν with θ=0. No CP violating Majorana phases exist in our mass matrix.

The simplest choice of $\theta = 0$ provides the case where the real part of M_ν respects the μ-τ symmetry. This texture has been discussed in Ref.[13, 17], which takes the form of

$$M^{\mu-\tau}_\nu = \mathrm{Re}\begin{pmatrix} M_{ee} & M_{e\mu} & -\sigma M_{e\mu} \\ M_{e\mu} & M_{\mu\mu} & M_{\mu\tau} \\ -\sigma M_{e\mu} & M_{\mu\tau} & M_{\mu\mu} \end{pmatrix} + i\mathrm{Im}\begin{pmatrix} 0 & M_{e\mu} & \sigma M_{e\mu} \\ M_{e\mu} & M_{\mu\mu} & 0 \\ \sigma M_{e\mu} & 0 & -M_{\mu\mu} \end{pmatrix}, \tag{20}$$

where the real part is the well-known μ-τ symmetric texture as expected while the imaginary part breaks it.[2] The mixing angles of $\theta_{12,13}$ are given by

$$\begin{aligned}\tan 2\theta_{12} &\approx 2\sqrt{2}\frac{\mathrm{Re}\left(M_{e\mu}\right)}{\mathrm{Re}\left(M_{\mu\mu}\right)-\sigma\mathrm{Re}\left(M_{\mu\tau}\right)-\mathrm{Re}\left(M_{ee}\right)},\\ \tan 2\theta_{13}e^{i\delta} &= 2\sqrt{2}\sigma\frac{i\mathrm{Im}\left(M_{e\mu}\right)}{\mathrm{Re}\left(M_{\mu\mu}\right)+\sigma\mathrm{Re}\left(M_{\mu\tau}\right)+\mathrm{Re}\left(M_{ee}\right)},\end{aligned} \tag{21}$$

from Eqs.(16) and (17). The expression of $\tan 2\theta_{12}$ is obtained by taking the approximation $\sin^2\theta_{13}\approx 0$. The maximal CP violation by $e^{i\delta}=\pm i$ is explicitly obtained.

Summarizing our discussions, we have advocated to use the possibility that the real part of M_ν only respects the μ-τ symmetry. This possibility is extended to the more general case of $M_\nu = M_{+\nu}+M_{-\nu}$ in Eq.(18), where $M_{+\nu}$ serves as a μ-τ symmetric texture and the symmetry-breaking term of $M_{-\nu}$ acts as a source of $\sin\theta_{13}\neq 0$. The consistency of the texture is given by the property that particular combinations of z, z^* and $e^{i\theta}$ become real or pure imaginary. This property ensures the appearance of real values of $\theta_{12,13}$ while the real value of θ_{23} arises from $\tan\theta_{23}=\mathrm{Im}(\mathbf{M}_{e\mu})/\mathrm{Im}(\mathbf{M}_{e\tau})$. It should be noted that θ_{23} is not determined by $\tan\theta_{23}=-\mathrm{Re}\left(\mathbf{M}_{e\tau}\right)/\mathrm{Re}\left(\mathbf{M}_{e\mu}\right)$ as in the μ-τ symmetric texture because the Dirac CP violation phase is now active. It turns out that $M_\nu = e^{i\theta/2}[\mathrm{Re}(e^{-i\theta/2}M_\nu)+i\mathrm{Im}(e^{-i\theta/2}M_\nu)]$, which gives no intrinsic Majorana CP violation while the Dirac CP violation becomes maximal.

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